Inference for the Progressively Type-I Censored Step-stress Accelerated Life Test under Interval Monitoring

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OBJECTIVES

In this work, we considered a step-stress accelerated life test under progressive Type-I censoring when a continuous monitoring of failures is infeasible but inspections at particular time points is possible. In addition to the accelerated failure time model to explain the effect of stress changes, a general scale family of distributions was considered for flexible modeling by allowing different lifetime distributions at different stress levels. When the inspection points align with the stress-change time points, the maximum likelihood estimators of the scale parameters and their conditional density functions could be derived explicitly. If the inspection points do not align with the stress-change time points, the parameter estimates can be obtained numerically.

1. Obtain and compare the MLEs
2. The exact CIs, the botstrap CIs and the approximate CIs
3. Compare the simulation results

MODELS

The design of the experiment is shown as in Figure 1.

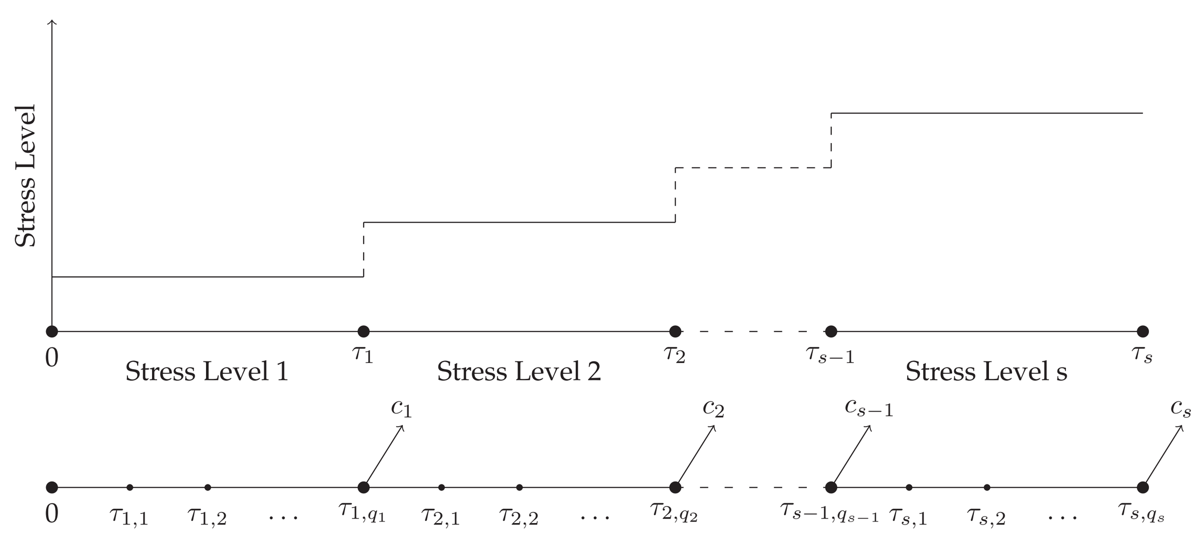


Figure 1: SSALT under progressive Type-I censoring with interval monitoring and intermediate inspection points

The likelihood is

$$L \propto \sum_{i=1}^{s}\sum_{j=1}^{q_i} n_{ij} \log\left(e^{-\frac{g_i(\tau_{i,j-1})}{\theta_i}} - e^{-\frac{g_i(\tau_{i,j})}{\theta_i}}\right) + \sum_{i=1}^{s} n_{i+}\frac{g_i(\tau_{i-1})}{\theta_i} - \sum_{i=1}^{s}(N_i - n_{i+})\frac{g_i(\tau_i) - g_i(\tau_{i-1})}{\theta_i} \quad (1)$$

METHOD

The maximum likelihood estimator can be obtained by solving the following equations, (2) is the likelihood equation for the case with the intermediate inspection points and (3) is the likelihood equation for the case without the intermediate inspection points.

$$\sum_{j=1}^{q_i} n_{ij}\frac{g_i(\tau_{i,j-1})e^{-\frac{g_i(\tau_{i,j-1})}{\theta_i}} - g_i(\tau_{i,j})e^{-\frac{g_i(\tau_{i,j})}{\theta_i}}}{e^{-\frac{g_i(\tau_{i,j-1})}{\theta_i}} - e^{-\frac{g_i(\tau_{i,j})}{\theta_i}}} = n_{i+}g_i(\tau_{i-1}) - (N_i - n_{i+})(g_i(\tau_i) - g_i(\tau_{i-1})) \quad (2)$$

$$\hat{\theta}_i = \frac{g_i(\tau_i) - g_i(\tau_{i-1})}{log(N_i) - log(N_i - n_i)} \quad (3)$$

In Table 1, some particular choices of distribution functions are shown.

Table 1: Different types of $g(t)$ fucntions

$g(t)$	Support	Distribution
t	$(0, \infty)$	Exponential
$t^\delta, \delta > 0$	$(0, \infty)$	Weibull($\delta = 2$: Rayleigh)
$log(1 + \frac{t}{d}), d > 0$	$(0, \infty)$	Lomax($d = 1$: special beta of second kind)
$e^{dt} - 1$	$(0, \infty)$	Gompertz

In the case of simple step-stress model ($s = 2$), the stochastic monotonicity of $P(\hat{\theta}_i > \xi | A)$ for $i = 1, 2$ can be proved. Figure 2 shows the increasingness of the MLEs.

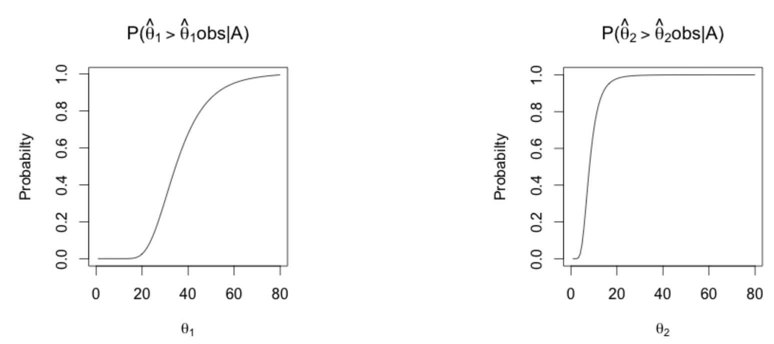


Figure 2: Increasingness of the MLEs for the data in Table 4

Therefore the $(1 - \alpha)\%$ exact confidence interval for θ_1 and θ_2 can be derived by solving (4) and (5) respectively.

$$\sum_{\left\{(i,j)\in A: \frac{g_1(\tau_1)}{log(n) - log(n-i)} > \hat{\theta}_{1obs}\right\}} \frac{\binom{n}{i}\binom{n-i-c_1}{j}(1-p_1)^{c_1}{p_1}^i{p_2}^j{p_3}^{(n-i-j-c_1)}}{P(A)} = \beta \quad (4)$$

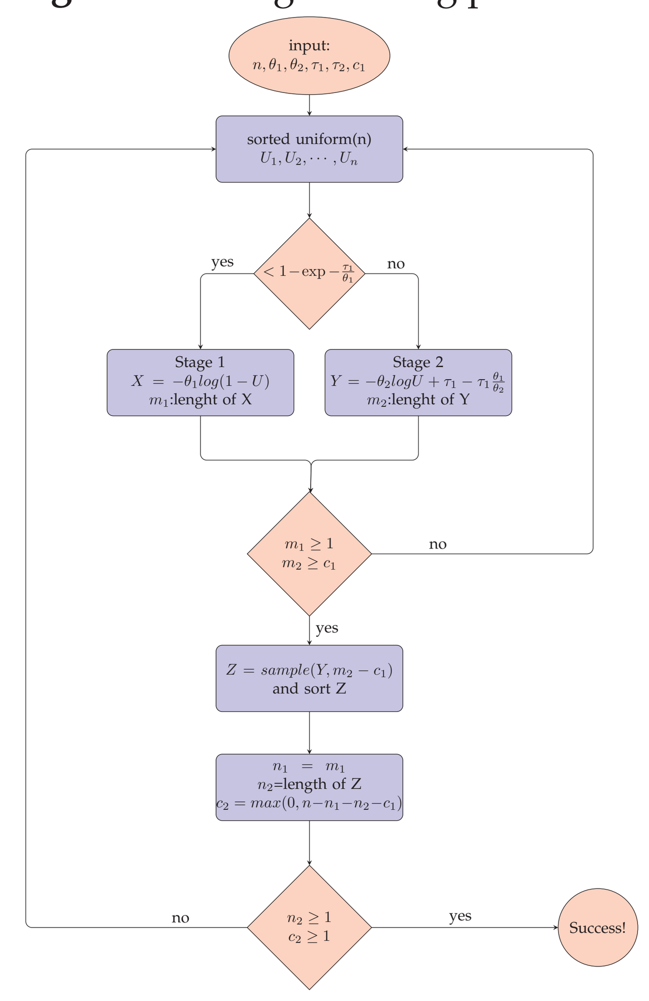
$$\sum_{\left\{(i,j)\in A: \frac{g_2(\tau_1) - g_2(\tau_1)}{log(n-i-c_1) - log(n-i-c_1-j)} > \hat{\theta}_{2obs}\right\}} \frac{\binom{n}{i}\binom{n-i-c_1}{j}(1-p_1)^{c_1}{p_1}^i{p_2}^j{p_3}^{(n-i-j-c_1)}}{P(A)} = \beta \quad (5)$$

where $\beta = \frac{\alpha}{2}$ for the upper-bound and $\beta = 1 - \frac{\alpha}{2}$ for the lower-bound.

SIMULATION & RESULTS

The precedure to generate the data is shown as in Figure 3.

Figure 3: Data generating procedure



The simulation are based on 1000 Monte Carlo simulations with $n = 20$, $\theta_1 = 12.18125$, $\theta_2 = 4.4817$, $c_1 = 2$, $\tau_1 = 5$, $\tau_2 = 9$ and $R = 1000$ bootstrap replications for each simulation. The results are listed in Table 2 and Table 3.

Table 2: Average values and standard deviations [in brackets] of the MLEs of $\theta_1 (= 12.18125)$ and $\theta_2 (= 4.4817)$

	Interval	Continious	MSD
$\hat{\theta}_1$	14.0949 [6.8634]	14.0673 [6.8202]	0.3658
$\hat{\theta}_2$	4.9617 [2.5964]	4.9886 [2.5705]	0.2437

Table 3: Estimated coverage probabilities(in%) of the CIs of θ_1 and θ_2 and mean lengths of the CIs

	90% CI		95% CI		99% CI	
	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
Coverage probabilities						
Exact CIs	0.900	0.933	0.923	0.962	0.990	0.996
Bootstrap BCa CIs	0.870	0.913	0.911	0.973	0.957	0.984
Approximate CIs	0.795	0.805	0.900	0.8450	0.960	0.872
Mean lengths						
Exact CIs	20.583	11.495	23.842	17.451	27.021	26.910
Bootstrap BCa CIs	28.225	9.731	35.244	12.455	53.843	19.165
Approximate CIs	15.508	5.480	18.628	6.293	25.519	7.976

FUTURE RESEARCH

- The log-link model.
- Optimal progressively Type-I censored step-stress accelerated life test under interval monitoring.

ILLUSTRATION

A two-level step-stress test was conducted under progressive Type-I censoring in order to assess the reliability characteristics of a solar lighting device. The experiment data are listed in Table 4.

Table 4: Progressively Type-I censored dataset from $n = 30$ prototypes of a solar lighting device

Stress Level	Failure	Times					
Level 1	1.515	2.225	4.629	4.654	6.349	8.003	$n_1 = 11$
	8.262	10.416	11.381	12.433	14.755		$c_1 = 4$
Level 2	15.164	15.355	15.953	16.735	18.796	19.248	$n_2 = 7$
	19.295						$c_2 = 8$

The maximum likelihood estimates of the parameters of θ_1 and θ_2 for all the data sets in Table 4 are givin in Table 5.

Table 5: MLEs for the interval monitoring and MLEs for the continuous monitoring[in bracket] and 95% CIs for $\hat{\theta}_1$ and $\hat{\theta}_2$

	θ_1	θ_2
MLE	32.8401[33.6020]	7.9541[7.9351]
Exact	(19.9494 66.9836)	(4.0161 19.1822)
Bootstrap BCa	(19.6814 67.2213)	(3.7453 19.0575)
Approximation	(17.3905 48.2897)	(3.6768 12.2313)

CONCLUSION

The interval monitoring can be an option if the continuous monitoring is impossible.

- Our estimates are competitive in terms of accuray.
- The exact CI performs better than approximate CI and boostrap CI in terms of coverage probability.

REFERENCES

P.BOBOTAS, M.KATERI. *The step-stress tampered failure rate model under interval monitoring, Statistical Methodology (2015) 100-122.*

D.HAN. *Time and Cost Constrained Optimal Designs of Constant-stress and Step-stress Accelerated Life Tests, Reliability Engineering and System Safety (2015) 1-14.*